

A Z score can also be derived using Newton's iterations and equation 4 as

follows:

Define  $P = \phi(z)$ , and then from equation 4:

$$\phi(z) \cong 1 - \left(1 - \frac{1}{2z^2}\right) \frac{1}{z\sqrt{2\pi}} e^{-z^2/2}$$

$$\text{let } G(z) = \phi(z) - p$$

Substituting the expression of  $\phi(z)$  into  $G(z)$  results in equation 5:

$$\begin{aligned} \text{EQN (5)} \quad G(z) &= 1 - p - \left(1 - \frac{1}{2z^2}\right) \frac{1}{z\sqrt{2\pi}} e^{-z^2/2} \\ &= \xi - \left(1 - \frac{1}{2z^2}\right) \frac{1}{z\sqrt{2\pi}} e^{-z^2/2} \end{aligned}$$

Taking the first derivative of  $G(z)$  with respect to  $z$  results in:

$$\frac{dG(z)}{dz} = \frac{d\phi(z)}{dz} = \phi'(z)$$

Hence, the iteration formula is given by equation 6:

$$\text{EQN (6)} \quad Z_{n+1} = Z_n - \frac{G(Z_n)}{\phi'(Z_n)} \quad n=0, 1, 2, 3, 4, \dots$$